Impact of Subsidies on Evolution of Fertility and Human Capital in the Iranian Economy*

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Abstract

On the contrary to common explanations of fertility transition, in the Iraninan Economy, there has been a growing human capital without decline of fertility for some decades followed by a rapid fall in fertility without a higher growth of human capital during the next decades. Accounting for these facts, we propose a new theoretical link between subsidies financed by exogenous oil revenue and the evolution of fertility and human capital. While the educational subsidy accelerates the growth of human capital, the presence of financial subsidies keep fertility from declining. Then, fertility declines with sizeable delay but sharply because (i) high fertility in first generations implies a growth rate of population more than the growth rate of oil revenue and therefore, lowers the subsidy per capita in next generations; (ii) by increasing the real wage through human capital evolution, the income effect of financial subsidies on fertility would decrease; and finally (iii) educational subsidy brings about more access to education, less human capital dispersion, less differential fertility, and consequently higher average of human capital and less fertility rate.

1 Introduction

Two interrelated facts in the Iranian economy during recent decades need explanation. Total fertility rate in one hand fell sharply from 7 births per women in 1979 to 1.9 in 2006. On the other hand, human capital has been growing steadily while both the inequality of human capital and differential fertility were decreasing during the past decades.

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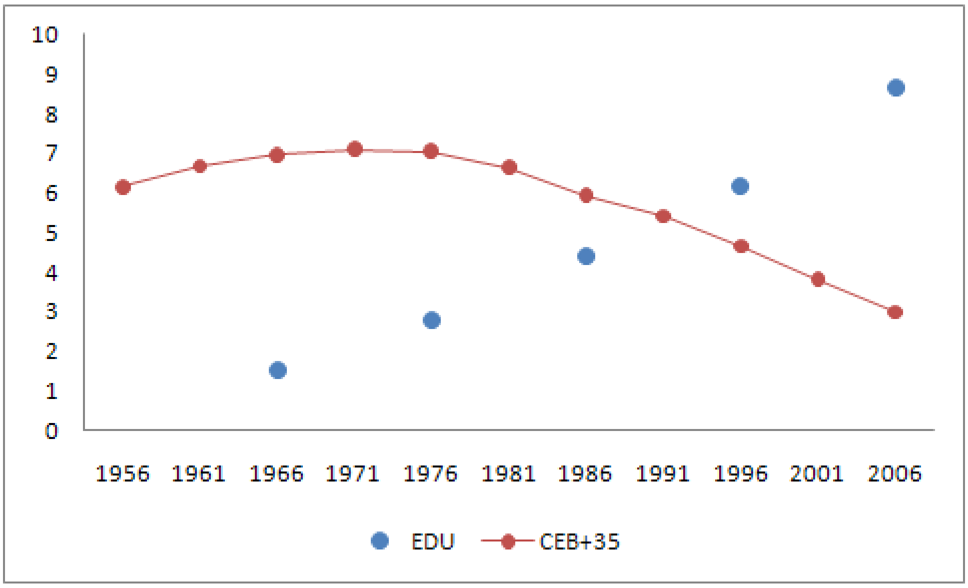

Figure 1: average Iranian women CEB & average Iranian workers' years of schooling

Figure 1 shows the evolution of fertility and educational attainment since more than the last half century. In each point, CEB shows the number of children ever born for the women who are in the same cohort.[1] We shift CEB from the year of women's birth by 35 years to make the number of adults' children comparable to their period of working. The average CEB for women born during 1922 to 1951 is between 6 and 7 while it falls for the cohort born in 1967-71 to below three.[2] Meanwhile, the average years of schooling of the Iranian workers has been increasing steadily from 1.5 in 1966 to 8.6 in 2006.

On the other hand, both human capital inequality and differential fertility has been decreasing. Figure 2 shows the "standard deviation of CEB shifet by 35 years" and "gini of workers' schooling years" in past decades. standard deviation of CEB has decreased from 2.83 for cohort born in 1931 to 1.78 for cohort born in 1961. On the other hand, gini of workers' schooling years has decreased from 0.83 in 1966 to 0.46 in 1996. The decrease in STD(CEB+35) has accelerated from 1986 (cohort born in 1951) but the steep of Gini(Edu) is approximately constant.

Figure 3 shows the frequency distribution of Children Ever Born for the Iranian women in different cohorts who have completed (or almost completed) their fertility plan. This figure illustrates the evolution of fertility distribution from older to younger cohorts. The mode

[1] Here by CEB we mean the number of children born *alive* to a woman.

[2] Since we concern about the number of children women had after completing their fertility plans, CEB (Children Ever born) is more appropriate than TFR (Total Fertility Rate) for the purpose of this research. Moreover, in this stage we do not have access to detailed Census data in 2006 to illustrate the distribution of fertiltiy here. So figure 1 is prepared based on detailed fertility data in 1996. Besides, the last average CEB reported here belongs to women who have been 39 years in 2006. Although the reported statistics of youngest cohort need some more years to capture the completed fertility, relevant estimations/predictions for completed fertility of current female generation suggest a number below three and even close to two (Abbasi-Shovazi et al (2009)).

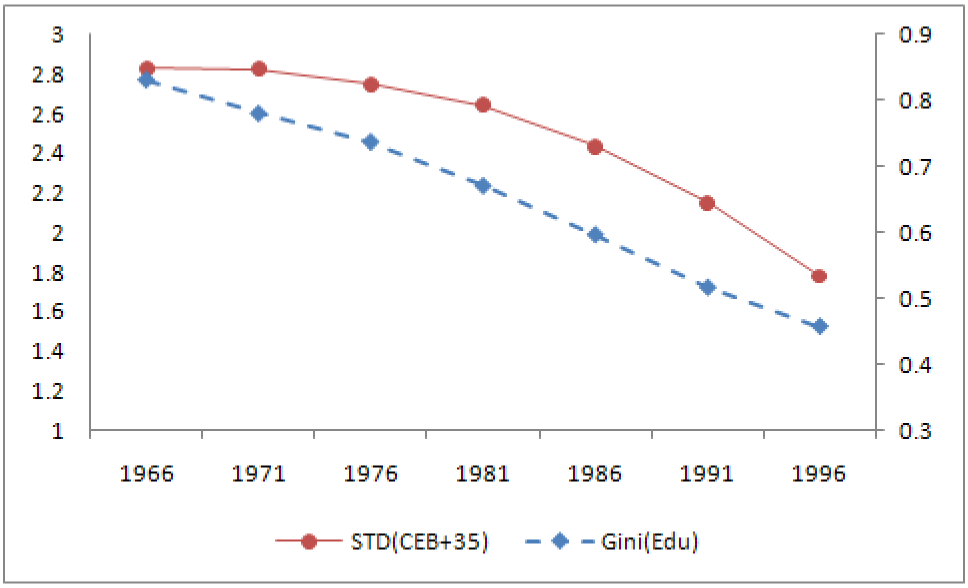

Figure 2: standard deviation of CEB shifted by 35 years (STD(CEB+35)) & gini of workers' schooling years (Gini(Edu))

of distributions which is 8 for the oldest cohort gradually decreases to 3 for the youngest cohort. From older to younger cohorts' distributions, the mean and the mode are decreasing (related to the first moment), the dispersion is compressing (related to the second moment), and the positive skewness is increasing (related to the third moment).

On the other hand, figure 4 shows the distribution of educational attainment of workers in 1966, 1986, and 2006. Based on Census data (gathered by Statical Center of Iran) five levels of education are considered: illiterate, primary, lower secondary, upper secondary, and tertiary. In 1966, 71 percent of workers are illiterate while the share of workers with tertiary education is close to zero. Then, we observe a considerable change in the distribution of educational attainment on workers just during four decades: in 2006, only 13 percent are illiterate while the shares of other education levels (primary, lower and upper secondary, and tertiary) are 10-20 percent higher than 1966 [3].

The common wisdom of fertility transition states that the more human capital parent have, the more income they earn, the higher opportunity cost of time they bear, the higher opportunity cost of raising children they face, and consequently the fewer children they make. On the contrary, in the Iranian Economy, one observes a growing human capital without decline of fertility for some decades followed by a rapid fall in fertility without a higher growth of human capital during next decades.

In this paper, we account for these interdependent facts by proposing a new link between *subsidies financed by exogenous oil revenue* and *the evolution of population and human capital* in the context of Iran's economy.

In the Iranian Economy the government has historically supported households' consumption and investment in education by widespread subsidies. During 1997 to 2008, the share of

[3] Nili-Nafisi (2005) and authors' calculation, both based on SCI Cencus data in 1966, 1986, and 2006)

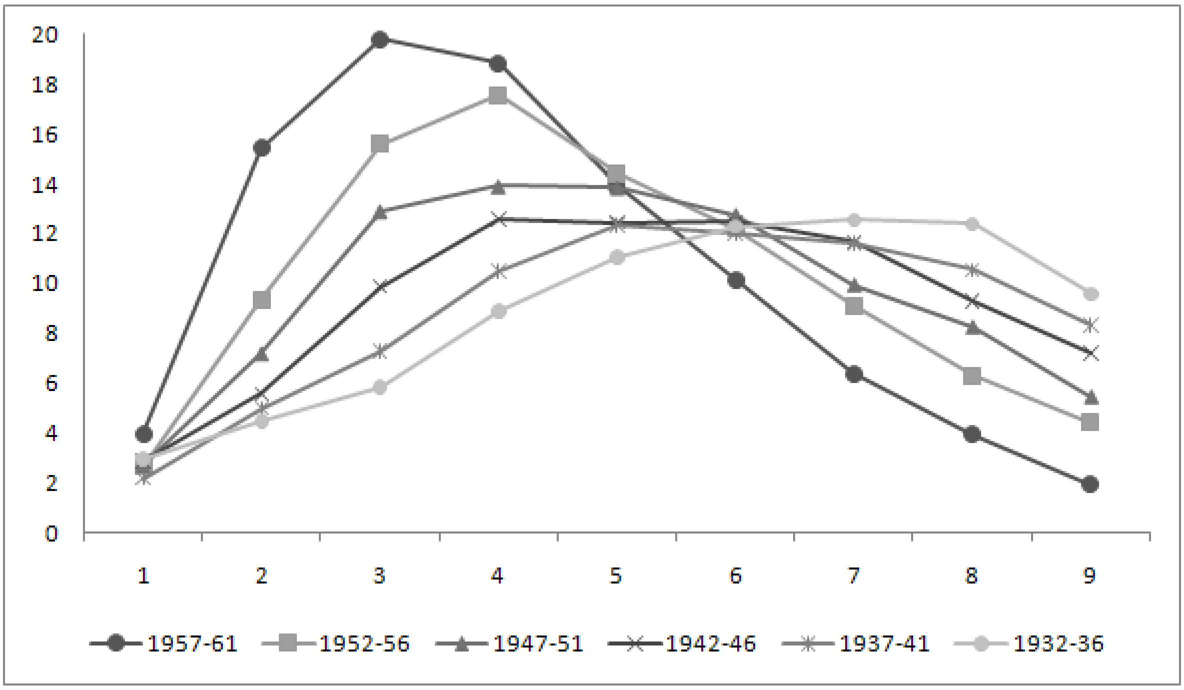

Figure 3: Distribution of CEB for Iranian women in different cohorts. Authors' calculation based on SCI fertility data in 1996)

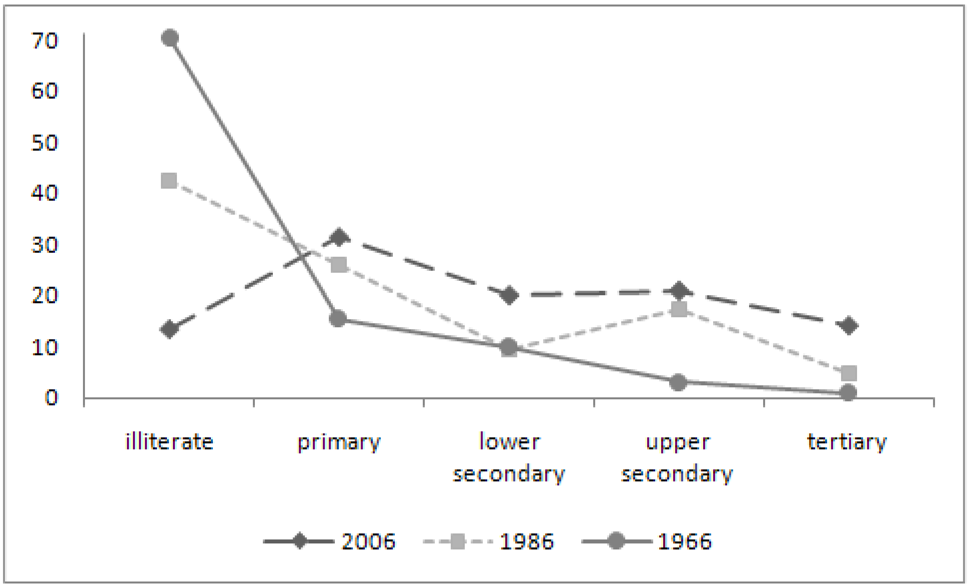

Figure 4: Distribution of educational attainment of Iranian workers (Nili-Nafisi (2005) and author's calculation based on SCI Census data in 1966, 1986, 2006)

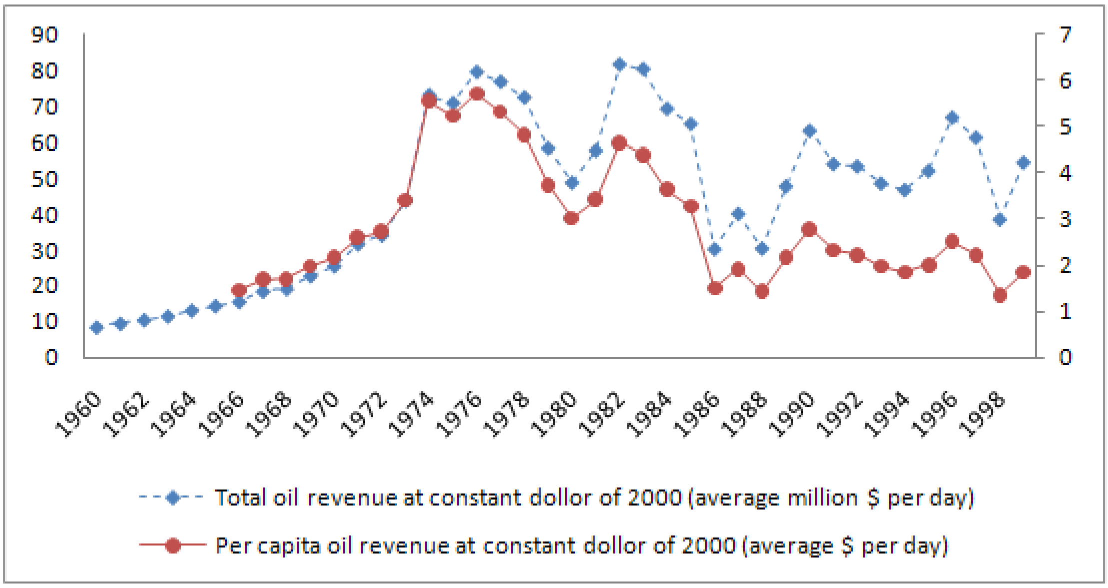

Figure 5: Iran's real total oil revenue & oil revenue per adult– at constant dollor of 2000 (U.S. Energy Information Administration)

budegtary funds on subsidies with respect to current government expenditure was on average 12.7 percent; while the share of subsidies allocated to basic consuming goods with respect to total budgetary subsidies according to GFS data provided by IMF was 60-70 percent (IMF GFS Data). Moreover, especially in post-revolutionary era, there have been large amount of subsidies as extra-budgetary funds which are mainly classified in credit, foreign exchange, and domestic energy consumption. These hidden expenditures matter for private consumption since the first one lowers the saving and two latter ones proivde consuming goods at lower prices. The shares of these subsidies to GDP have been changed through the time though remained significant in many years (Salehi-Esfahani and Taheripour (2000)).

On the other hand, for some decades, the provision of education was dominated by public sector while the share of private sector education has gradually increased in recent decades. Even in 2006, the ratio of public expenditure on education to GDP was 5.1 percent and the ratio of public expenditure on education to total government expenditure was 19.5 percent (United Nations Common Database).

These subsidies have been financed by oil revenues in addition to tax revenues. During 1995 to 2006, the average share of total oil revenues of the government with respect to the total government's revenue was 61 percent[4].

Figure 5 illustrates the real total oil revenue and the per adult oil revenue in Iran's economy. Real oil revenue increased with a remarkable growth in 1960s and 1970s (from 20 to 80 million dollor per day at constant dollor of 2000); and after some fluctuations in late 1970s and early 1980 reached to 30 million dollor per day at constant dollor of 2000; then

[4]Based on the new form of budget data reporting by the Central Bank of IR Iran, total oil revenue can be obtained by the sum of four figures: oil revenue, reserve withdrawal, profit from National Iranian Oil Company, and tax on National Iranian Oil Company

fluctuated in a range of 40-60 million dollor per day at constant dollor of 2000 in 1990s. On the other hand, oil revenue per adult (20-64 years old) has increased exponentially from 1 to 6 dollor per day at constant dollor of 2000 from 1960s to mid-1970s. Arriving 1980s oil revenue per capita has decreased considerably due to both reduction in total oil revenue and prior high population growth (to approximately 1.5 dollor per day at constant dollor of 2000 in 1990s).

Regarding the significant presence of subsidies financed by oil on one hand, and the questionable interrelated pattern of fertility and human capital in Iran's economy on the other hand, in this paper we explore some theoretical aspects of the effects of financial and educational subsidies on households' decisions to show how their aggregate behavior might make a channel in accounting for the evolution of fertility and human capital in Iran.

On link to the literature, we concentrate on the milestones which are relevant to our work. The early literature falls in microeconomics of family behavior followed by models of economic growth concerning human capital as the engine of growth and fertiltiy as an endogenous variable. The complementary studies are reuqired to investigate the impacts of government's subsidies on economic growth.

Becker and Lewis (1973) analyze parents' quantity-quality trade-off in their decision on children, given the parents' income and relative prices. Further studies which matter for our purpose fall in the investigation of the aggregate behavior of households.

Since presenting the role of fertility choice in a model of economic growth by Barro and Becker (1988), and introducing human capital as the engine of growth by Lucas (1988), there has been an expanding literature on the link between fertility, human capital, and growth. Lucas proposed an external effect of human capital represented by the average level of human capital in addition to an internal effect of an individual's human capital on individuals' productivity.

The evolution of a growing economy is typically described by a transition from high fertility and low human capital (stagnation equilibrium) to a low fertility and high human capital steady-state (Becker, Murphy & Tamura (1990) and Galor & Weil (2000)).

Galor and Tdisson (1997) analyze the interaction between human capital distribution and economic growth stating that besides average level of human capital, composition of human capital contributes as a significant factor in accounting for the process of development. In their model, the individual's human capital is an increasing function of both the level of parental human capital and the average level of human capital of the economy.

De la Croix and Dopeke (2003) have accounted for the U-shaped relation between growth and inequality by proposing a new channel through differential fertility. The higher income (or human capital) inequality in their model leads to wider differential fertility because poor and rich decide differently for quantity and quality of their children. Consequently in the next generation, the rise of workers with lower quality leads to less human capital on average and lesser growth. The significance of differential fertility in accounting for different phases of economic development and inequality is also supported by Ehrlich and Kim (2007).

The literature supports the idea that analysis of the process of development including fertility transition and human capital evolution demands for recognizing the distributions of both fertility and human capital over the development process.

Regarding the basic lines of analysis by Becker and Lewis (1973) and interaction between human capital distribution and economic growth by Galor and Tdisson (1997), we augment de la Croix-Doepke model by incorporating a government that benefits from oil revenues and supports households by different subsidies. Government revenue is composed of tax revenue plus an exogenously determined flow of oil revenue. On the expenditure side the government allocates its budget to public spending and subsidizing private consumption and education.

At the family level, the availability of subsidies financed by oil revenues changes the conventional solution of quantity-quality problem. The *financial subsidy* on the contrary to the wage income imposes no opportunity cost of time to parents while increases their income. Consequently the financial subsidy brings about more fertility especially for the parents with less human capital whose incomes increase by a higher margin due to a certain amount of financial subsidy. The *educational subsidy*, by reducing the price of schooling, brings about more access to education for children. Therefore, on one hand it increases schooling years of children in rich enough households, and on the other by lowering "the educational poverty threshold" enables some poor households to have educated children.

Given the initial distribution of fertility and educational attainment, the model combines three effects in accounting for the evolution of population and human capital in an oil abundant developing country like Iran. First and second, the effects of financial subsidy and educational subsidy on households' decisions; and third, the way that differential fertility makes role in tracing the aggregate behavior of the economy.

The paper is organized in five sections. In section 2 the model economy is presented. In section 3 we analytically solve households' problem under the presence of different subsidies which explains how the dynamic aggregate behavior of households in the presence of government budget constraint could shape the pattern of fertility and human capital in Iran's economy. In section 4 we provide a relevant computational experiment to replicate the stylized facts of data in a quantitative manner. Finally section 5 concludes.

2 The Model

There are consecutive generations at $t = 0, 1, 2,...$ in an economy in which each person lives for two periods: childhood and adulthood. At each generation decisions are made by single-parent adults. Only adults consume while children benefit from attaining human capital by schooling. In the initial generation, there are N_0 adults who are distributed I groups according to their level of human capital. We sort these I groups in such a way that the level of human capital in group $i + 1$ is higher than group i. In the first generation, there is an initial discrete distribution of human capital on these groups. We trace the evolution of population and human capital through these generations while total population N_t, is sum of the population of all groups i.e. $N_t = \sum_{i=1}^{I} N_t^i$ for $t = 0, 1, 2...$.

For the sake of simplicity, we abstract from physical capital which leads to abstract from financial market including savings as supply of financial resources, capital as an input of firms, interest rate as the equilibrium of the financial market, and also the presence of old-age. Moreover, we abstract from sorting and matching in marriage and assume that each person decides on quantity and quality of her children by herself while becoming an adult.

To examine households' decisions regarding substitution effects, at first, we take a general case of households' problem. Then we make some assumptions to be more specific on the form of utility function and the relative prices.

Each adult enjoys from her consumption, and quantity and quality of children. In equation (i), c, n, e respectively stands for adult's consumption, number of children, and educatinal attainment of children which is assumed to be the same for all the children of a family.[5] Utility function of an adult in group i at t is described as follows

$$(i) : U^i = U(c^i, n^i, e^i)$$

where U is strictly increasing and concave with respect to its three arguments:

$$U_c > 0, U_n > 0, U_e > 0, U_{cc} < 0, U_{nn} < 0, U_{ee} < 0$$

An adult's time is allocated to child-parenting and working while total effective time is normalized to 1. Each child takes ϕ units of parent time, so abstracting form $(1 - \phi n)$ is the time allocated to work. We also standardize the wage of each unit of human capital to 1, i.e. a representative adult's wage is a linear function of her level of human capital. So wage income of an adult with human capital level of h is $(1 - \phi n)h$. Moreover each single-parent receives z amount of financial transfer.

Household's expenditures are consumption and investing in education of children. In equation (ii) e_t is the level of education that each child receives where the constant price of each extra year of schooling is p^e. Moreover, each child imposes the living cost of p^n to the parent. Equation (ii) describes the budget constraint of an adult in group i as:

$$(ii) : c^i + p^n n^i + p^e n^i e^i = (1 - \phi n^i)h^i + z^i$$

Decision occurs in a point that the ratio of marginal utility of the quantity of children (n) to marginal utility of their quality (e) equals the ratio of marginal cost of n to marginal cost of e:

$$(iii) : \frac{MU_n}{MU_e} = \frac{\phi h + p^n + p^e e}{p^e n}$$

Marginal cost of n originates from forgone income of allocating time to care one extra child (ϕh), living cost of one extra child (p^n), and cost of investing in the human capital of one extra child $(p^e e)$. Marginal cost of e is the cost of one more level of schooling of n existing children $(p^e n)$. Equation (iii) captures the possible substitution effects between the quantity and quality of children with no restricted assumption on functional form of utility and relative prices.

Based on equation (iii) and concavity of U with respect to its arguments, increasing ϕ (child-care time-cost) results in less n and/or more e. In addition, increasing the financial subsidies targeted households' consumption lowers p^n and consequently brings about more fertility. It is worth mentioning that financial transfer z_t has no substitution effect on the quantity and quality trade-off.

[5]Since in the absence of sorting in marriage, each adult decides on n by herself, we can regard $2n$ as if it is the number of children of a couple. We use this point in comparison of simulated CEB and actual CEB in section 4.

Here, we briefly examined how relative prices affect the solution of the quantity-quality problem in a general case. We postpone the complete solution to section 3 and for this stage, we make some specific assumptions on the form of utility function and the prices, and then introduce other parts of the model.

We assume that utility function takes a simple log version as:

$$U_t^i = ln(c_t^i) + \gamma ln(n_t^i h_{t+1}^i) \tag{1}$$

where $\gamma > 0$ is the altruism factor and h_{t+1} stands for the stock of human capital of a child who becomes an adult in generation $t+1$. Parents enjoy not only from their own consumption, but also from the quantity n_t and quality h_{t+1} of their children.

Moreover, we assume the teachers' wage equals the average wage of working adults in the society, i.e. teachers' human capital equals the average level of human capital of the economy. Although all households face the same price of schooling, they bear different cost of caring children as we assume p_t^n for an adult in group i is proportionate to her human capital, i.e. $p_t^n = \rho h_t^i$ where $0 < \rho < 1$, i.e. ρ represents children's marginal propensity to consume as a share of parent's human capital.

Each household receives three types of subsidies:

1. The educational subsidy that loweres the price of schooling by u_t.
2. The financial transfer equals to z_t which increases the income of a household without imposing any cost of time. We assume that each household receives constant amount of z_t. The presence of z_t captures the income effect on fertility while the substitution effect is captured by the other part of income originated from labor market. The income effect would increase fertility while the substitution effect would decrease it.

Rewriting equation (ii) regarding points made above results in:

$$c_t^i + \rho h_t^i n_t^i + \overline{h}_t(1-u_t)n_t^i e_t^i = (1-\phi n_t^i)h_t^i + z_t \tag{2}$$

The human capital evolves through generations as:

$$h_{t+1}^i = H(\mu + e_t^i)^\alpha (h_t^i)^\beta (\overline{h}_t)^\delta \tag{3}$$

This shows that the stock of human capital of an adult in generation $t+1$ (h_{t+1}^i) depends on the length of schooling she received (e_t^i), the stock of her parent's human capital (h_t^i), and the average level of human capital of parents in the economy ($\overline{h}_t$).

The parameter H is the scale of efficiency of human capital transfer. Parameter μ captures the out-of-schooling process of gaining human capital. Here we assume that besides schooling, some parts of human capital is gained from the interaction between non-schooling parameter (μ) and adults' human capital (h_t), and between non-schooling parameter (μ) and the average human capital of the economy ($\overline{h}_t$). Parameters α, β, and δ represent respectively return to education, the effect of home externality of interaction with parents, and the effect of spill-over of human capital in the society. All these parameters take values in $(0,1)$ interval. Furthermore, presence of $\overline{h}_t$ in the equation of human capital transfer could be interpreted as the quality of education. Moreover, to have an endogenous growth, we assume $\beta + \delta = 1$.

There is a labor market in the economy where L_t represents the aggregate effective labor supply:

$$L_t = \sum_{i=1}^{I} N_t^i h_t^i (1 - \phi n_t^i) \tag{4}$$

We assume that the labor market clears and labor demand equals labor supply. There is only one firm in the economy that employs L_t as the input of its production function:

$$Y_t = A L_t \tag{5}$$

where Y_t is the aggregate production or equivalently the national income of the economy. The parameter A is the scale of efficiency of the aggregate production function which is normalized to 1.[6]

The average level of human capital is given by:

$$\overline{h}_t = \sum_{i=1}^{I} \frac{N_t^i h_t^i}{N_t} \tag{6}$$

Total population of consecutive generations evolves as:

$$N_{t+1} = \sum_{i=1}^{I} n_t^i N_t^i \tag{7}$$

The population share of group i in generation t is given by $\frac{N_t^i}{N_t}$. The cumulative distribution function of human capital $F_t(h_t^i)$ evolves through generations through $t = 1, 2, ...$ for heterogeneous agents of the economy classified in I groups $(i = 1, 2, ..., I)$ as:

$$F_t(h_t^i) = \sum_{j=1}^{i} \frac{N_t^j}{N_t} \tag{8}$$

Finally, there is a government who has access to oil endowment and supports households with educational subsidy and financial transfer. We abstract from all other public spending

[6] National Income Identity can be obtained by aggregation on households' budget constraint (equation 2) and government's budget constraint (equation 9):

$$\sum_{i=1}^{I} N_t^i c_t^i + \sum_{i=1}^{I} N_t^i \rho h_t^i n_t^i + \sum_{i=1}^{I} N_t^i \overline{h}_t (1 - u_t) e_t^i n_t^i = \sum_{i=1}^{I} N_t^i (1 - \phi n_t^i) h_t^i + \sum_{i=1}^{I} N_t^i z_t^i$$

$$\Longrightarrow \{\sum_{i=1}^{I} N_t^i c_t^i + \sum_{i=1}^{I} N_t^i \rho h_t^i n_t^i\} + \sum_{i=1}^{I} N_t^i \overline{h}_t e_t^i n_t^i = Y_t + \Gamma_t$$

$$\Longrightarrow C_t + I_t^{education} = Y_t + \Gamma_t$$

In our model, NII is described as: aggregate private consumption of the adults and children plus aggregate investment equals aggregate income (productin) plus exogenous oil revenue.

and simply assume that educational subsidy and transfers are the only expenditures that the governmnet incur. Furthermore, we abstract from tax revenue and assume that there is no budget deficit and in each period the exogenous amount of government revenue Γ_t which is financed by oil sale equals to sum of expenditures on different subsidies:

$$\Gamma_t = \sum_{i=1}^{I} N_t^i n_t^i e_t^i \overline{h_t} u_t + N_t z_t \tag{9}$$

Definition: Partial Equilibrium

Given an exogenous amount of government revenue Γ_t, an initial distribution of human capital $F_t(h_t^i)$, and an initial size of adults population N_t, a *partial equilibrium* of the economy is defined as a sequence of decision variables $\{c_t^i, n_t^i, e_t^i\}_{i=1}^I$, the aggregate quantities $\{L_t, Y_t, \overline{h}_t, N_{t+1}\}$, a distribution $F_{t+1}(h_{t+1}^i)$, and a set of policy variables $\{u_t, z_t\}$ such that:

- given the set of policy variables $\{u_t, z_t\}$, decision variables $\{c_t^i, n_t^i, e_t^i\}_{i=1}^I$ solve the maximization problem of each household, i.e. the maximization of (1) subject to (2) and (3);
- aggregate quantities $\{L_t, Y_t, \overline{h}_t, N_{t+1}\}$ satisfy equations (4), (5), (6), and (7);
- the distribution of human capital $F_t(h_t^i)$ evolves as (8); and finally
- policy variables $\{u_t, z_t\}$ satisfy (9).

3 Theoretical Solution

In this section, we provide analytical solution of the model indicating the partial equilibrium of the economy. At family level, we trace the ways that households' decisions on consumption, and quantity and quality of children are affected by the financial and educational subsidies. Then based on our theoretical findings, we explain how the dynamic aggregate behavior could shape the pattern of fertility and human capital consistent with the stylized facts of the Iranian economy.

We define q_t^i as the relative human capital of an adult in group i at t as:

$$q_t^i = \frac{h_t^i}{\overline{h}_t} \tag{10}$$

Households' problem is identified by maximization of (1) subject to (2) and (3). Putting (3) in (1), the households' problem for $i = 1, 2, ..., I$ is:

$$\max_{c_t^i, n_t^i, e_t^i \geq 0} ln(c_t^i) + \gamma ln(n_t^i(\mu + e_t^i)^\alpha) + const.$$

$$s.t. : c_t^i + \rho h_t^i n_t^i + \overline{h}_t(1 - u_t)e_t^i n_t^i = (1 - \phi n_t^i)h_t^i + z_t$$

In consistency with equation (iii) the relative marginal utilities equals to relative prices at:

$$\frac{MU_n}{MU_e} = \frac{\gamma(\frac{1}{n})}{\gamma\alpha(\frac{1}{\mu+e})} = \frac{\mu+e}{\alpha n}$$

$$= \frac{h(\phi+\rho)+\overline{h}e(1-u)}{\overline{h}n(1-u)}$$

These equations indicate that the relative marginal utility of n to e is equivalent to $(\mu+e)/\alpha n$. Higher return to education 'α' brings about less marginal utility of n to e and consequently more schooling, i.e. a substitution effect for the quality. In the same manner, more less ρ has a substitution effect in favour of the quality while more educational subsidy, u, has a substitution effect for the quantity.

Solving households' problem leads to:

$$c_t^i = \frac{h_t^i + z_t}{1+\gamma} \tag{11}$$

This indicates that in addition to individual's human capital, the financial transfer, z_t, has a direct income effect on consumption.

For $q_t^i \geq T_t^E = \frac{\mu(1-u_t)}{\alpha(\phi+\rho)}$, there is an interior solution for n and e as:

$$n_t^i = \frac{\gamma(1-\alpha)}{1+\gamma}\frac{q_t^i + (z_t^i/\overline{h}_t)}{q_t^i(\phi+\rho)-\mu(1-u_t)} \tag{12}$$

$$e_t^i = \frac{1}{1-\alpha}(\frac{\alpha(\phi+\rho)q_t^i}{1-u_t} - \mu) \tag{13}$$

For $q_t^i < T_t^E = \frac{\mu(1-u_t)}{\alpha(\phi+\rho)}$, there is a corner solution:

$$n_t^i = \frac{\gamma}{(1+\gamma)(\phi+\rho)}(1+\frac{z_t}{h_t^i}) \tag{14}$$

$$e_t^i = 0 \tag{15}$$

The minimum level of relative human capital in generation t is q_t^{min} which is greater than zero because fertility for an adult with zero human capital goes to infinity based on equation (12.2), so we set a non-zero minimum level for q_t.

The complete solution of households' problem on n and e in compact forms are:

$$n_t^i = Max\{\frac{\gamma(1-\alpha)}{1+\gamma}\frac{q_t^i + (z_t^i/\overline{h}_t)}{q_t^i(\phi+\rho)-\mu(1-u_t)}, \frac{\gamma}{(1+\gamma)(\phi+\rho)}(1+\frac{z_t}{h_t^i})\}$$

$$e_t^i = Max\{\frac{1}{1-\alpha}(\frac{\alpha(\phi+\rho)q_t^i}{1-u_t} - \mu), 0\}$$

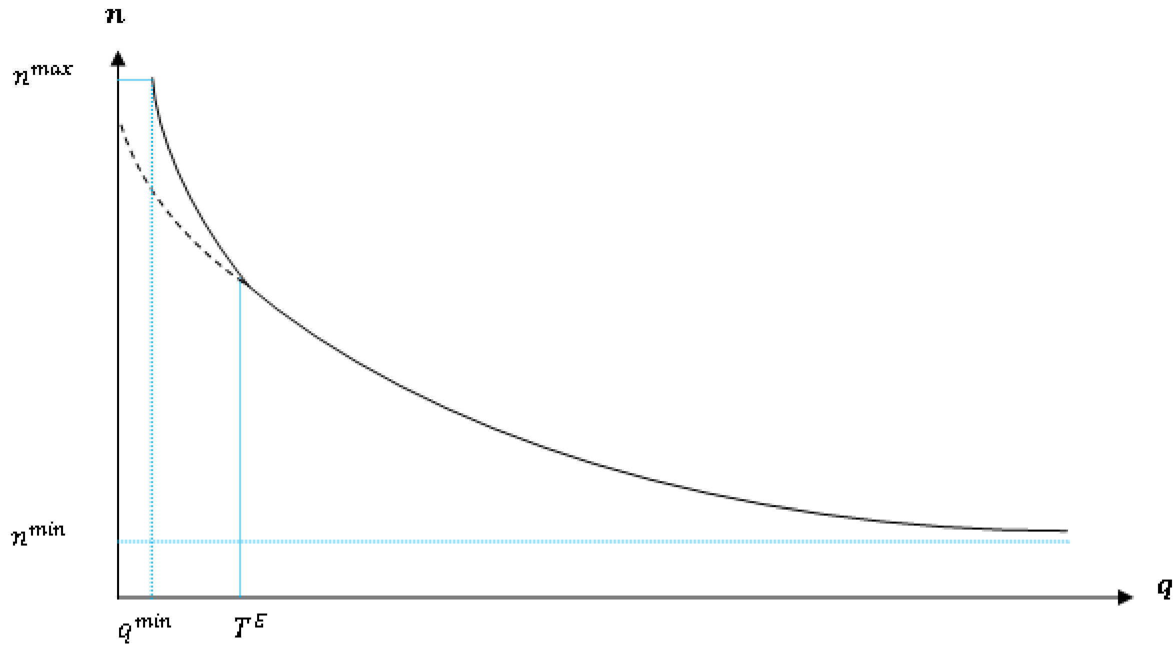

Figure 6: number of children versus relative human capital

We call T^E "the educational poverty threshold" because adults with relative human capital lesser than this threshold do not send their children to educational institutions; in other word these households bear educational poverty.

For rich enough households, i.e. where $q_t^i \geq T_t^E$, since the price of schooling is the average level of human capital, the real wealth effect of financial transfer is a function of $z_t/\overline{h}_t$. On the other hand, for poor enough households, i.e. where $q_t^i < T_t^E$, that the schooling is zero, the real wealth effect of financial transfer is a function of z_t/h_t^i. Considering the fact that h_t^i is lower than $\overline{h}_t$ for poor enough households, the real wealth effect of z_t on the fertility of the poor would be greater. In the presence of large enough financial transfer, and given an initial distribution of human capital with large enough density at poor families, this point could be a channel of maintaining fertility at a high level for the first generations whose human capital dispersion is relatively higher than subsequent generations.

Other things being equal, figure 6 shows the number of children versus relative human capital.

Following de la Croix-Doepke model, the trade-off between quantity and quality of children is captured in the model as:

$$\frac{\partial n_t}{\partial q_t} < 0 \tag{16}$$

$$\frac{\partial e_t}{\partial q_t} > 0 \tag{17}$$

As we concern about the impacts of financial and educational subsidy on fertility, following inequalities show the partial derivatives of fertility with respect to different subsidies and child-care cost:

$$\frac{\partial n_t}{\partial z_t} > 0 \tag{18}$$

$$\frac{\partial n_t}{\partial \rho} < 0 \tag{19}$$

$$\frac{\partial n_t}{\partial u_t} < 0 \tag{20}$$

Inequalitiy (18) indicates that fertility is an increasing function of z_t and inequality (19) indicateds that it is a decreasing function of ρ. The relation (20) states that more educational subsidy rate causes less fertility. This latter result needs explanation. In our model, although there is no saving, households act as if they are investing all of their savings on their childrens' human capital. In this context, they enjoy from two types of alternatives: from consumption and quantity of children, and from investment on children. This indicates that more educational subsidy rate leads to lower relative cost of investment and equivalently higher relative price of making children, and consequently fewer children.

Schooling is an increasing function of educational subsidy as:

$$\frac{\partial e_t}{\partial u_t} > 0 \tag{21}$$

Everything being equal, higher educational subsidy rate causes the educational poverty threshold to shift to the left in figure (6):

$$\frac{\partial T_t^E}{\partial u_t} < 0 \tag{22}$$

Non-schooling could be an optimal decision for poor enough households because even at the level of no-schooling, the marginal benefit of attaining human capital from the spill-over of human capital, i.e. the interaction between $\overline{h}_t$ and μ, would be more than the net marginal benefit of schooling. Higher educational subsidy rate lowers the price of schooling and so decreases the marginal cost of schooling. Therefore, some households which had been in the left side of the educational poverty line would fall to the right side of the line and get rid of the educational poverty trap.

Inequalities (18), (19), (20), and (21) indicate that more educational subsidy, u, shifts the diagram of figure 6 to the left while more financial subsidy, z, pushes the diagram upward.

For differential fertility, i.e. the ratio of n^{max} to n^{min} we have:

$$DF = \frac{n^{max}}{n^{min}} \simeq \frac{1 + z/h^{min}}{1 - \alpha} \tag{23}$$

We use approximation sign here because h^{min} is close to greater than zero while n^{max} is obtained for a zero human capital. The term n^{max}/n^{min} can be regarded as a simple index for Differential Fertility. This result indicates that everything being equal, more financial transfer, z, increases DF. Meanwhile, less human capital of the poor adults heightens DF. The interpretation of the effect of z/h^{min} on DF is straight-forward: the income effect of a certain amount of financial transfer on fertility is higher for poorer adults.

Based on latter result, and given a considerable human capital dispersion among households, differential fertility would be higher in the presence of financial transfers. This could explain the high differential fertility in the initial generation of our model. Moreover, considering the fact that high enough educational subsidy increases human capital of children in both poor and rich households, differential fertility would decrease in next generations, which again has been the case in the economy of Iran.

In the followings, based on the above equations we explain how our findings could partially account for the evolutionary pattern of fertility and human capital in the economy of Iran[7].

Consider an economy where initially distribution of human capital is characterized by a low average and relative high dispersion of human capital with a greater density at the poor. In the first generations, the educational subsidy heightens the growth of human capital (consider relations 13, 21, and 22) but the presence of financial subsidy and financial transfer keeps fertility from declining (consider relations 12, 14, and 18). Fertility, then, falls with sizeable delay but sharply due to following reasons:

1. Because of high fertility of the previous generations, population has increased far faster than the growth of oil revenue so subsidy per capita decreased considerably.

2. Due to human capital evolution, the real wage has increased (consider relation 3) so that income effect of financial subsidies on households decision on fertility has decreased (consider realations12 and 14).

3. Thanks to the educational subsidies, access to education has been more equalized (consider relations 13, 21, and 22) and consequently human capital dispersion has declined which causes less differential fertility, higher average of human capital and less fertility rate (as suggested by de la Croix-Dopeke (2003)).

4 Quantitave Examination

In this section, we provide a computaional experiment to show the quantitative aspects of the model. We have faced two kinds of difficulties for quantitative examination of the model. First, there are some common parameters, such as ϕ, which we could not find for the underlying economy in previous studies. Second, the required data of government's subsidies and financial supports, due to the significant presence of hidden expenditure (Salehi-Esfahani & Taheripour (2000)), absence of detailed official figures in some old years and frequent change of classifications and definitions are in some cases difficult to obtain.

[7] At least two other variables matter for elaboration of the fertility transition in Iran: First, the mortality rate, which has been steadily decreased from 217 per thousand births in 1951 to 25 in 2006 (Abbasi et al (2009)). Second, the short-run baby boom in the late 1970s and early 1980s; which is observed by TFR but not CEB suggesting a model with endogenous marriage decision. In this paper, we abstract from both mortality and marriage decision since we are concern about CEB not TFR and are accounting for not all features of fertility transition in Iran, but one questionable part of its long-run pattern.

Considering these restrictions, our quantitave experiment is not supposed to be regarded as measurements with high degree of accuracy. Rather, our purpose of this numerical exercise is to illustrate how our theoretical results might quantitatively generate the main features of the pattern of fertilty and human capital in Iran's economy.

Calibration

Reasonable predictions indicate that fertility rate in Iran will drop to the reproduction rate in future years (Abbasi et al (2009)). Thus following De la Croix-Doepke (2003) altruism factor γ can be set to 0.271 which in the absence of effective subsidies makes the growth rate of population in balanced growth path equals to the reproduction rate.

The parameter α affects the elasticity of human capital (in our model equivalent to wage or earning) with respect to schooling. Based on Mincer equation, estimated elasticity of wage to schooling equals to 0.069, 0.088, and 0.076 respectively for 1987, 2001, and 2006 (Salehi-Isfahani (2009)). Since 1987 lies between the older and younger generations in our model, we choose 0.069 as the elasticity of wage to schooling for all generations. The elasticity of wage to schooling is a function of μ and α. Assuming μ as equivalent to 0.012 as the benchmark model, the implied α is 0.5.

The parameter ϕ in De la Croix-Doepke's model is calibrated to 0.075 estimated in the empirical studies on US economy suggesting that the opportunity cost of a child is equivalent to about 15 percent of the parents' time endowment. In the Iranian Economy with respect to the US economy, female labor participation rate is singinficantly lower (in 2008 is 31.2 percent for Iran in comparison with 58.9 percent for US (UN Data)); and the labor market is less efficient. Therefore, as a relevant figure, we set ϕ to 0.040 which is rather half of the benchmark model's calibration. While ϕ is classified as opporunity cost, ρ is financial cost of one extra child with respect to parent's earning. We set $\rho = 0.035$. Non-educational cost of children is $(\phi + \rho)h_i$. In our calibration $(\phi + \rho)$ equals to 0.075 which is as the same as ϕ in de la Croix-Doepke's calibration (their model does not include ρ).

The parameter β does not affect the households' decisions though it influences the growth rate. We choose $\beta = 0.5$ and standardize $A = 1$ while set the efficiency scale H as the annual growth rate of Y_t is set to 4.7 percent (equals to the average annual growth rate of real GDP in Iran during the longest period that Macro data are documented).

Table 1: Calibrated Parameters

parameter	**sign**	**quantity**
Altruism factor	γ	0.271
Return to education	α	0.5
Intergeneration human capital transfer factor	β	0.5
Constant term of human capital transfer	μ	0.012
Time-cost of child-care	ϕ	0.04
Marginal child's propensity to consume	ρ	0.035

Initial Distribution of Human Capital

According to Census data in 1966, we access to the distribution of educational attainment of workers in 5 levels of education (illiterate, primary, lower secondary, upper secondary, and tertiary). Based on these 5 education levels, we set $I = 5$.

Regarding the fact that in our model there is a one-to-one correspondence between schooling of children e_t and relative human capital of parents q_t (except for $q_t^i < T_t^E$) we can find the initial distribution of human capital for the adults who lived in the predecessor generation of 1966. i.e. the adults in 1931 if we take childhood as 20 years and adulthood as 30 years and assume that workers in 1966 are in the middle age of their adulthood (20+15 years). This idea benefits us in two ways: first, we can go to one generation before what the first Census data explicitly reports; second, it appears a hidden variable (human capital) derived from an observable variable (schooling years).

Seventy percent of illiterate workers in 1966 implies seventy percent of adults who have a relative human capital ($q_{t_0}^1$) less than educational poverty line ($T_{t_0}^E$). We set $q_{t_0}^1$ equals to $kT_{t_0}^E$ for $0 < k < 1$ and choose k close to one for current examination. Consequently, the initial distribution of (relative) human capital are obtained as in Table 2.

Table 2: Initial distribution of relative human capital

group i	1	2	3	4	5
q^i	0.19	2.02	3.41	4.78	6.62
Density	0.71	0.15	0.10	0.03	0.01

Profile of Policy Variables

We assume that in each period, the government knows the amount of oil revenue, Γ_t, average level of human capital, $\overline{h_t}$, size of each groups' population, N_t^i for $i = 1, 2, ..., I$, and households' decision making. The latter information indicates that for a vector of parameters, $\Omega = [\alpha, \phi, \gamma, \mu, \rho]$, n_t^i and e_t^i are functions of u_t and z_t:

$$n_t = n(u_t, z_t; \Omega)$$

$$e_t = n(u_t, z_t; \Omega)$$

Rewriting governments' budget constraint leads to:

$$\Gamma_t = \sum_{i=1}^{I} N_t^i n^i(u_t, z_t; \Omega) e^i(u_t, z_t; \Omega) \overline{h}_t u_t + N_t z_t$$

This equation is not sufficient for simultaneous specification of u_t and z_t. Hence, we assume that the government first sets u_t, then based on the aforementioned equation, z_t is obtained. We set u in a way that the educational system is semi-public in t_2, more public in t_1, and less public in t_0. Based on the history of oil revenue, the ratio of oil revenue in t_1 and t_2

respect to t_0 is calculated. We set Γ_0 in a way that the ratio of Γ_0 to Y_0 equlas the ratio of public spending to gross domestic prodcution in t_0.

Table 3 shows government's revenues and the profile of policy variables.

Table 3: Profile of policy variables

ploicy varaible	z_t/z_0	u_t	Γ_t/Γ_0
t_0	1	0.4	1
t_1	2.25	0.6	8.5
t_2	0.49	0.5	13

Simulation and Results

We run a computational experiment to illustrate how quantitative results of our model could simulate the main long-run features of the evolution of fertility and human capital in the Iranian Economy.[8]

Consistent with the initial distribution of human capital, the initial generation becomes adult in 1931 and lives for 30 years followed by the next generation. We simulate the model for three generations: $t_0 = 1931, t_1 = 1961, t_2 = 1991$ where t_i shows the year that a generation becomes adult.

The results of the simulation on fertility are shown in table 4. Since n is the quantity of children for a single parent, $2n$ shows the quantity of children as for a couple. Therefore, $2n_t^1$ and $2n_t^5$ respectively shows the fertility in groups 1 (the poorest) and 5 (the richest) while $2n_t$ shows the average fertility of all households. The figures are consistent with the common fact that the more human capital parents have, the fewer children they make. On the other hand, the results capture the dynamics of fertility in Iran: from t_0 to t_1 fertility declines only slightly from 6.89 to 6.09 (a delay in declining of fertility), but from t_1 to t_2 it declines sharply from 6.09 to 3.76.

Table 4: Results on Fertility

generation	$2n_t^1$	$2n_t^5$	$2n_t$
t_0	8.49	2.93	6.89
t_1	6.55	2.90	5.09
t_2	3.82	2.86	3.76

Table 5 shows the results for the inequalities: $DF_t = n_t^1/n_t^5$, $Gini_t^e$, and $Gini_t^h$ respectively show the ratio of fertility of the poorest to the richest, gini coefficient of education, and gini coefficient of human capital. The result captures the decline in differential fertility and educational attainment. During $[t_0, t_2]$, DF decreases from 2.90 to 1.34; gini coefficient

[8]It is worthwhile to emphasize that our quantitive experiment is trial and it can be enhanced to be more accurate in future steps of development of this incomplete version of the paper.

of education decreases from 0.77 to 0.67; and gini coefficient of human capital decreases from 0.62 to 0.57. Educational attainment inequality decreases steadily while human capital inequality decreases with a delay. Reduction in educational attainment inequality in t_1 makes a channel for human capital inequality to decrease in t_2.

Table 5: Results on Inequalities

generation	$DF_t = n_t^1/n_t^5$	$Gini_t^e$	$Gini_t^h$
t_0	2.90	0.77	0.62
t_1	2.26	0.74	0.65
t_2	1.34	0.67	0.57

5 Concluding Remarks

The evolution of fertility and human capital in Iran is described by an initial high fertility and low human capital. Fertility declines with a sizeable delay but sharply while human capital has been growing steadily. Meanwhile, both differential fertility and human capital inequality have been decreasing continuously. We propose here a new channel in which different subsidies financed by exogenous oil revenues affect the parents' decisions on the quantity and quality of children and consequently in its turn, shape the dynamics of population and human capital evolution.

We augment the model proposed by de la Criox and Doepke (2003) with a government who receives exogenous oil revenues as part of its revenue and besides public spending, supports the households by three types of subsidies: educational subsidy, financial subsidy aiming at private consumption, and financial transfer. In the first stage of development, the educational subsidy heightens the growth of human capital, but the financial subsidies keep fertility from declining.

The sharp delayed decline of fertility in our model is captured through three channels: (i) High fertility in first generations results in a growth rates of population more than the growth rate of oil revenue and expenditures on subsidies. Therefore, the subsidy per capita decreases in subsequent generation; (ii) Inter-generational human capital transfer (due to home externality and spill-over of human capital in the society besides schooling) increases the real wage. Financial tranfer affects fertility proportionate to individuals' or average human capital. Therefore, evolution of human capital decreases the income effect of financial transfer on fertility; (iii) Educational subsidy as a policy which is targeting more access to education, brings about less human capital dispersion and consequently less differential fertility. Following the benchmark model, less differential fertility causes a higher average of human capital and less fertility rate in the next generation.

In the first place, this paper could be regarded as a theoretical effort to capture the represented ideas. Moreover, under a relevant profile of policy variables, the quantitative results confirms our theoretical findings. This work might be followed by more concentarion on the quantitative study on determinants of fertility and educational attainment in the Iranian Economy.

References

[1] Abbasi-Shavazi, Mohammad Jalal and McDonald, Peter and Hosseini-Chavoshi, Meimanat, *The Fertility Transition in Iran: Revolution and Reproduction*, Springer, (2009).

[2] Bassetto, Marco, *Political economy of taxation in an overlapping-generations economy*, Review of Economic Dynamics 11 (2008).

[3] Becker, Gary S. and Barro, Robert J., *A Reformulation of the Economic Theory of Fertility*, Quarterly Journal of Economics, (Feb., 1988).

[4] Becker, Gary S. and Murphy, Kevin M. and Tamura, Robert, *Human Capital, Fertility, and Economic Growth*, Journal of Political Economy, (Oct., 1990).

[5] Becker, Gary S. and Lewis, H. Gregg, *On the Interaction between the Quantity and Quality of Children*, Journal of Political Economy (1973).

[6] Bloom, David E. and Williamson, Jeffrey G., *Demographic Transition and Economic Mirecales In Emerging Asia*, NBER Working Paper Series, Working Paper 6268 (Nov., 1997).

[7] CastellA, Amparo and DomA nech, Rafael, *Human Capital Inequality and Economic Growth: Some New Evidence*, Economic Journal, Conference Papers (Mar., 2002).

[8] de la Croix, David and Doepke, Matthias, *Inequality and Growth: Why Differential Fertility Matters*, American Economic Review, (Sep., 2003).

[9] de la Croixa, David and Doepke, Matthias, *Public versus private education when differential fertility matters*, Journal of Development Economics 73 (2004).

[10] de la Croixa, David and Doepke, Matthias, *To Segregate or to Integrate: Education Policies and Democracy*, NBER Working Paper Series, Working Paper 13319 (Aug., 2007).

[11] Ehrlich, Isaac and Kim, Jinyoung, *The Evolution of Income and Fertility Inequalities over the Course of Economic Development: A Human Capital Perspective*, Journal of Human Capital, (2007).

[12] Galor, Oded and Moav, Omer, *From Physical to Human Capital Accumulation: Inequality and the Process of Development*, Review of Economic Studies, (Oct., 2004).

[13] Galor, Oded and Tsiddon, Daniel, *The Distribution of Human Capital and Economic Growth*, Journal of Economic Growth, (March 1997).

[14] Galor, Oded and Weil, David N., *Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and beyond*, American Economic Review, (Sep., 2000).

[15] Glomm, Gerhard and Ravikumar, B., *Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality*, Journal of Political Economy, (1992).

[16] Jones, Larry E. and Schoonbroodt, Alice and Tertilt, Michele, *Fertility Theories: Can They Explain The Negative Fertility-Income Relationship?*, NBER Working Paper Series, Working Paper 14266 (Aug., 2008).

[17] Jones, Larry E. and Tertilt, Michele, *An Economic History of Fertility in the U.S.: 1826-1960*, NBER Working Paper Series, (Nov., 2007).

[18] Mincer, Jacob A., *Introduction to Schooling, Experience, and Earnings*, NBER Working Paper Series, Working Paper 1762, (1974).

[19] Nafisi S., Nili M., *The Relation Between Human Capital and Economic Growth with Emphasize on the Role of Education Distribution of Labor Force: Case of IRAN 1966 2000*, MSc Thesis, Graduate School of Management and Economics, Sharif University, (2005).

[20] Salehi-Isfahani, Djavad and Tunali, Insan and Assaad, Ragui, *A Comparative Study of Returns to Education of Urban Men in Egypt, Iran, and Turkey*, (Version Oct., 2009)

[21] Salehi-Isfahani, Djavad , *Population, Human Capital, and Economic Growth in Iran*, Human capital: population economics in the Middle East, Chapter 6, (2003).

[22] Salehi-Esfahani, Hadi and Taheripour, Farzad, *Hidden Public Expenditures and Economic Performance in Iran*, Version prepared for the International Journal of Middle Eastern Studies (Sep., 2000).

[23] Stafford, George E. and Stafford, Frank P., *On the Rate of Return to Schooling Quality*, Review of Economics and Statistics, (Nov., 1996).

Statistics:

`www.imf.org/external/pubs/ft/gfs/manual/gfs.htm`, IMF Government Finance Statistics (GFS).

`http://amar.sci.org.ir/PlanList.aspx`, Statical Center of Iran, Census Data in 1956, 1966, 1976, 1986, 1996, 2006, and Fertility Survey Data in 1996.

`http://www.cbi.ir/Category/EconomicTrends_en.aspx`, Journal of Seasonal Economic Trends, Central Bank, IR IRAN.

http://tsd.cbi.ir, Economic Time Series Database, Central Bank, IR IRAN.

http://unstats.un.org/unsd/default.htm, United Nation Common Database

http://unstats.un.org/unsd/cdb_discontinued/cdb_discontinued.asp, United Nation Statistic Divisions