Voting Behavior in Multiple Round Elections: An Analysis of Iran's 2005 Presidential Election.

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Abstract

For the first time after 1979 revolution in Iran, the 2005 presidential election went to the second round. Not only neither candidates could win the majority of the votes in the first round, but the potential voters in the first round were fairly divided between the 7 candidates and abstention. It created a relatively rich menu of options for the voters to choose from, however by using data from election results I show that they voted (or abstained) strategically in the first round.

I present a model that justifies the possibility of the change in voting behavior between the first and second round elections in a two stage election game and I analyze the data to explore the possibility of strategic voting based on this change. I present tests to show that the results are still valid in spite of using aggregate data.Moreover the empirical result sheds light on the voting behavior of different factions within the society and their response to different political and ideological agendas.

1   Introduction

For the first time after 1979 revolution in Iran, the 2005 presidential election went to the second round. Not only neither candidates could win the majority of the votes in the first round, but the potential voters were fairly divided between the

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7 candidates and abstention. It created a relatively rich menu of options for the voters to choose from, however by using data from election results I show that they voted (or abstained) strategically in the first round. I present a model that justifies the possibility of the change in voting behavior between the first and second round elections in a two round election with many candidates. Then I use the data for the 2005 presidential election in the district level to analyze the voting behavior of the potential voters. I show that some voters who have voted for one of the candidates (Rafsanjani) in the first round voted for his opponent in the second round. Moreover some of those who abstained in the first round voted in the second round.

The model accounts for strategic voting or abstention in multiple rounds elections with many candidates, in which individuals might vote (abstain) in the primary round but abstain (vote) in the second round, otherwise subtraction of other candidates should not change the decision of the voter on whether to abstain or not. A more interesting aspect of strategic voting is that the voter might vote for a different candidate from the one that she voted in the previous round, while both options are present.

Following Brody and Page (1973) I distinguish between abstention due to alienation and abstention due to indifference. Abstention due to alienation happens for an individual who sees the candidates positions and characters too unattractive. I use a criteria for abstention due to alienation in multiple candidate elections, which is similar to what already exists. However, for abstention due to indifference, which is the result of the individual's indifference between the two candidates in the existing works (Riker and Ordeshook (1968,1973), Thurner and Eymann (2000), Adams et al (2006)), I present a new definition which is applicable to elections with many candidates . This new definition is based on sets of favorable and unfavorable candidates for a single voter. Results drawn from the presented voting and abstention model help to justify the change in voting behavior.

Elections with many candidates are interesting since there is a possibility of more than one round of voting [1], and it increases the possibility of strategic voting and abstention. The observed differences in the turn out between the two rounds could be a result of this strategic abstention. Using aggregate data for the 2005 presidential election, in which the election went to the second round between Ahmadinejad, the mayor of Tehran who was a young radical conservative, and Rafsanjani, a well known experienced politician who had eight years of presidency in his resume among

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[1]for the case of the primary elections in the US it happens with certainty.

many key positions after 1979 revolution, I show the extent of the change in voting behavior among potential voters. One interesting result is that there is a significant proportion of those who voted for Rafsanjani in the first round, voted for Ahmadinejad in spite of the presence of Rafsanjani in the second round. In addition, the decision to abstain in the second round among those who voted in the first round for different candidates is not the same.

The continuation of this paper is as the following: In the next section I present the model of voting. It is followed by the theoretical results in Section (3), and the empirical results in Section (4). Conclusion is presented in Section (5).

2 Model

Here I present a model of voting that extends the current models of voting and abstention to account for abstention in multiple candidate in multiple round elections. This model applies to elections whit majority rule, in which a candidate wins in the first round if she gets more than half of the votes casted, but the top two candidates compete in the second round ff neither candidates receive half of the total votes.

Suppose n individuals are eligible to participate in an election with a set of candidates I, and x_{ij} is a vector of the offered policies, together with character attractiveness of the candidate $j \in I$ for individual i. The utility gain of individual i from candidate j is summarized by $u_i(x_j)$. Individual i's perception about the probability of winning of the candidate j is denoted by p_{ij}, $0 < p_{ij} < 1$, with $\sum_{j \in I} p_{ij} = 1$. Moreover, Ω is the set of all nonempty subsets of I excluding I itself.

In each round the sequence of actions for the voter is as what Riker and Ordeshook (1973) suggested: First individuals decide whether to vote or abstain, and then choose the candidate for which to vote if they decided to vote. I extend the existing notion of abstention in two candidate elections - abstention due to alienation (Downs, 1957), and abstention due to indifference (Converse, 1966)- to multiple candidate elections. The definition of the abstention due to alienation is not different from the one used in two candidates models of election.

Definition 1. *An individual i abstain due to alienation if there is not any candidate j such that $u_i(x_j) > d_i$ for an individual specific d_i.*

The threshold d_i is different for different individuals and it depends on the individual preferences, or the cost of voting. This definition is similar to the ones given

for the two candidate elections, and the logic is simple: It does not matter how many candidates are running, if no one is attractive enough I don't vote.

The other reason for abstention is the voter's indifference between the candidates, even though some candidates are good enough. Applying this concept for the location models of elections with two candidates is easy, but the real challenge here is to define indifference in elections with multiple candidates. I use the following notion for abstention due to indifference.

Definition 2. *An individual i abstains due to indifference if there is no $w \in \Omega$ such that*

$$\sum_{j \in \omega} \frac{p_{ij} u_i(x_j)}{\sum_{j \in \omega} p_{ij}} - \sum_{j \notin \omega} \frac{p_{ij} u_i(x_j)}{\sum_{j \notin \omega} p_{ij}} \geq c_i \qquad (1)$$

for a given individual specific threshold c_i.

The idea is to divide the candidates in two groups, favorable and unfavorable, from a potential voter's point of view. The difference between the two groups of candidates must be big enough to give the voter incentive to vote. The first term in Equation (1) is the expected utility of the voter from the subset of the candidates ω, conditioned on one of them being an eventual winner. Similarly the second term is expected utility from a set of unfavorable candidates conditioned on a member of it being the winner. Then the inequality states that the difference of gains, in terms of expected utility, from the two sets of candidates must be bigger than an individual specific threshold. In two candidate elections, ω and its complement are singleton, and the notion of abstention is similar to Adams et al (2006).

The set ω may not be unique for multiple candidates elections. There might be many ωs that satisfies Condition (1). Suppose Φ is the set of all candidates belonging to at least one of those ωs, and $\bar{\omega}$ is the subset of ω that maximizes the left hand of (1). $\bar{\omega}$ is the set of candidate that differentiates the most between the set of favorable and unfavorable candidates. This set is useful in some results that is drawn in the next section.

I assume different criteria for voting behavior in the first and the second round voting. The voter votes for the candidate with the highest expected utility, $p_{ij} u_i(x_j)$, in the first round, and for the candidate who gives the highest utility, $u_i(x_i)$, in the second round. The difference makes sense since in the first round the voter might have several favorable candidates, and he has to consider the viability of the candidates as well as the utility gain from them. In the second round, however, the

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voter only picks the best among the two remained candidates, without being afraid of the threat of any other third candidate.

3 Theoretical Results

In this section some consequences of the above voting model are analyzed. The focus here is mostly on the voting behavior of a single voter and possible changes in voting strategies between the first and the second round of the election. The first two propositions are about abstention due to alienation.

Proposition 1. *If a voter abstains in the first round due to alienation , she will abstain again in the second round.*

The proof follows the definition of abstention due to alienation. Abstention in the first round means that the utility gain from any one of candidates is less than the voter's reservation threshold, and since the set of candidates in the second round is a subset of the set of first round candidates, any individual who abstains in the first round abstains in the second round as well. But the opposite is not true: from abstention in the second round if it is due to alienation one can not conclude that the voter should have abstained in the first round.

One might vote in the first round but abstain due to alienation in the second round if all js with $u_i(x_j) > d_i$ are eliminated in the first round. It follows the definition of abstention due to alienation. If abstention was only due to alienation, the immediate conclusion could be that the turn out must be always higher in the first round. However abstention might be because of indifference that the voter feels between the candidates. This notion is well developed in the two candidate setups, but it is more complicated in multiple candidate elections.

An individual might abstain in the first because of the indifference but vote in the second round because the contrast between the remaining candidates might increase by deletion of some of the candidates. Suppose $\bar{\omega}$ is the set that makes the most separation between the two groups of candidates (favorable and unfavorable), and the difference in conditional expectations is still below the threshold for voting. For this voter to vote in the second round the remaining candidates must be different enough to give her incentive to vote. The following proposition gives a limit on the extent of possibility of change in voting behavior from abstention in the first round to voting in the second round.

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Proposition 2. *An individual abstained in the first round due to indifference will not vote in the second round if neither of the two remaining candidates are from $\bar{\omega}$.*

The intuition for the proof follows from the fact that if $u_i(x_{il}) - u_i(x_{ik}) > c_i$ then by adding candidate l and all those candidates not in $\bar{\omega}$ which are better than candidate k to $\bar{\omega}$ we can construct a new set ω such that it satisfies condition (1).

Proposition 3. *An individual might vote for a candidate $j \in \omega$ in the first round but for another candidate $\acute{j} \in \omega$ in the second round if $\acute{j}$ runs against j.*

This situation happens if $u_i(x_{\acute{j}}) - u_i(x_j) > c_i$ but $f(x_j, p_{ij}) > f(x_{\acute{j}}, p_{ij})$. The reason is that the individual might prefer a candidate over others but perceives the candidate unlikely to get out of the first round. Hence the voter might vote for another candidate who is less favorable in the first round. In the second round, however, she votes for the candidate that she likes the most between the two remaining candidates. That is why some of the voters who voted for Rafsanjani in the first round might have changed their votes for Ahmadinejad in the second round, as is shown in the empirical results. Those are the voters who liked Ahmadinejad more but did not think that he has a significant chance of winning, so they voted for a less favorable but more viable candidate whom they still liked. The next proposition states the limit on the extent of change in vote.

Proposition 4. *In the second round, a voter will never vote for a candidate who is not in her $\bar{\omega}$ against a candidate in $\bar{\omega}$.*

The proof is presented in the appendix. This proposition states the usefulness of the set $\bar{\omega}$, in which maximizes the difference in expected payoffs of the two sets of favorable and unfavorable candidates. This set makes the biggest contrast between the two groups of candidates so that the voter will never vote for an unfavorable candidate against a candidate from the favorable group. The proposition shows another limitation of the extent that the voter might change her vote between the two rounds.

4 Empirical Results

4.1 The 2005 Presidential Election of Iran

The 2005 presidential election is unique in many ways and that gives the opportunity to test some aspects of the model of voting and abstention presented in Section (2).

In that election for the first time after 26 years of Islamic Republic in Iran, the presidential election went to the second round. The previous presidential elections were always won in the first round and often by landslide and the result was pretty much predictable long before the election with the exception of 1997 in which, Mohammad Khatami, a reformist candidate won but again in the first round and with a big margin. There were many other factors in the 2005 election which made it different and led to an almost unpredictable result. I use aggregate data from the 235 districts provided by the Ministry of Interior to analyze aspects of voting behavior.

The election primarily held between seven candidates: Rafsanjani who was the president for 8 years and a very influential figure in the last 30 years. He was believed to have a moderate view on social and political issues, being in favor of more market oriented economy and in favor of better relation with outside world. Ahmadinejad on the opposite was viewed as a new face among high ranking politicians. He was appointed as the governor of Ardabil province during Rafsanjani presidency and was the mayor of the capital city of Tehran at the time of election. He was known to have radical views especially in the standoff over nuclear issue. His economical stands was in favor of redistributive policies and his announced view on many social issues was moderate even though his main supporters among hardliners and Bassij was giving an opposite signal to many voters. The third candidate was Karrubi, the former head of parliament who was the deputy of the head when Rafsanjani held that position. He had close relation with the reformists, had a history of being hardliner in the first decade of the revolution and his main theme of the campaign was redistribution of the oil revenue by giving cash out cash. Karrubi and Rafsanjani were the only clergies and could make them popular among the old generation with traditional preferences. The fourth candidate was Larijani, former minister of culture and head of government run TV, a very outspoken critic of the reformist government and had roots in the conservative part of the society, and was known close to the supreme leader. The fifth candidate was Ghalibaf, a former commander in the revolutionary guard and the head of police, which could place him among more conservative candidates but being a pilot and having a good resume in security could make him an attractive candidate for the ordinary people. He was known to be the favorite candidate among Bassij as well but it seems the support shifted toward Ahmadinejad few days before the election. The sixth candidate was Moein a former minister of higher education and a reformist very close to the supporters

of the reformist president Khatami. He was popular among the university students. And the seventh candidate was Mehralizadeh, the head of the sports organization in the reformist government.

Data exhibits extensive variations for each candidate's vote share in the first round. As is shown in Table (1) each candidate has been in the first place in at least one district, mostly the districts in which they were born or have similar ethnicity, and each candidate has won a significant share of votes in many districts. That makes the data rich enough for the empirical results drawn in the next section.

Candidates	% Vote share	% Maximum	% Minimum	Districts > %10
Rafsanjani	22.0	68.1	2.6	300
Ahmadinejad	20.3	61.8	0.4	187
Karrubi	18.0	82.4	1.9	259
Moein	14.5	84.3	0.7	200
Ghalibaf	14.5	51.3	1.7	235
Larijani	6.1	74.7	0.6	41
Mehralizadeh	4.6	80.2	0.2	38

Table 1: Summary of the first round election results

4.2 Empirical Model

In this section I use the data from the 2005 Iran's presidential election in search of evidence of change in voting behavior between the two rounds of election, indicated by $t \in \{1,2\}$. In each district τ the number of votes that the candidate j received is shown by N_{jt}^τ. The total number of votes in district τ is $N_t^\tau = \sum_j N_{jt}^\tau$. Suppose the population eligible to vote in district τ is N^τ, then the number of those who abstained is $N_{Abt}^\tau = N^\tau - N_t^\tau$.

The set of candidates in the first round is $I = \{Ah, Ra, La, Gh, Mo, Ka, Me\}$, in which Ah stands for Ahmadinejad, Ra for Rafsanjani, La for Larijani, Gh For Ghalibaf, Mo for Moein, Ka for Karrubi, and Me for Mehralizadeh. The election went to the second round since none of the candidates received the majority of the vote. The set of candidates in the second round only includes Ahmadinejad and Rafsanjani because they received the highest number of votes in the first round.

The transition from the first to the second round is shown by $q_{jj'}$s. $q_{jj'}$ represents the proportion of those who voted for the candidate j in the first round and then

voted for the candidate $\acute{j}$ in the second round. $q_{j\acute{j}}$ can be viewed as the probability of a person who voted for the candidate j in the first round votes for the candidate $\acute{j}$ in the second round. In addition suppose q_{jAb} is the proportion of those who voted for the candidate j in the first round and then abstained in the second round, and q_{Abj} is the proportion of those who abstained in the first round but voted for one the candidates in the second round. Finally, q_{AbAb} is the proportion of those who abstained in the second round from those abstained in the first round.

The goal of this section is to estimate the parameters $q_{j\acute{j}}$ for all js and $\acute{j}$s, q_{jAb}, and q_{Abj} using the data from the 2005 Presidential Election in Iran published by Iran's Interior Ministry. We use the following system of equations to estimate these parameters.

$$N^{\tau}_{Ah2} = \sum_{j \in I} q_{jAh} N^{\tau}_{j1} + q_{AbAh} N^{\tau}_{Ab1} \tag{2}$$

$$N^{\tau}_{Ra2} = \sum_{j \in I} q_{jRa} N^{\tau}_{j1} + q_{AbRa} N^{\tau}_{Ab1} \tag{3}$$

$$N^{\tau}_{Ab2} = \sum_{j \in I} q_{jAb} N^{\tau}_{j1} + q_{AbAb} N^{\tau}_{Ab1} \tag{4}$$

These equations are identities, and the type of regression we use here is known as Goodman's regression (Goodman, 1953, 1959). Goodman's regression has long been used mostly to estimate the difference in voting behavior among different ethnicities using aggregate data. The problems related to using Goodman's regression has long been the discussion of many scholars, however it still has many applications, most notably in voting right cases after the U.S. Supreme Court endorsed its use (King, 1997).

To deal with the possible heteroskedasticity, because of population differences among the districts, I use the population adjusted equations, which are driven from the above equations by dividing them by the district population.

$$\frac{N_{Ah2}^{\tau}}{N^{\tau}} = \sum_{j \in I} q_{jAh} \frac{N_{j1}^{\tau}}{N^{\tau}} + q_{AbAh} \frac{N_{Ab1}^{\tau}}{N^{\tau}} \tag{5}$$

$$\frac{N_{Ra2}^{\tau}}{N^{\tau}} = \sum_{j \in I} q_{jRa} \frac{N_{j1}^{\tau}}{N^{\tau}} + q_{AbRa} \frac{N_{Ab1}^{\tau}}{N^{\tau}} \tag{6}$$

$$\frac{N_{Ab2}^{\tau}}{N^{\tau}} = \sum_{j \in I} q_{jAb} \frac{N_{j1}^{\tau}}{N^{\tau}} + q_{AbAb} \frac{N_{Ab1}^{\tau}}{N^{\tau}} \tag{7}$$

There are two restrictions that these equations need to satisfy to have meaningful implications for the model of voting behavior. The first restriction is that all q's should be in $[0, 1]$ interval. Moreover for each individual, regardless of her decision in the first round, has only three options in the second round: Vote for Ahmadinejad, vote for Rafsanjani or abstain. That implies

$$q_{jAh} + q_{jRa} + q_{jAb} = 1 \quad for \; all \; j \in I. \tag{8}$$

Moreover the options in the first round In Equation (5) the The results of the linear regression is reported below:

	Ahmadinejad	Rafsanjani	Abstention
Ahmadinejad	1.15	0.08⋆⋆[2]	-0.16
Rafsanjani	0.37	0.73	-0.10
Larijani	0.86	0.19	-0.05⋆⋆
Ghalibaf	0.78	0.13	0.09⋆[3]
Mehralizadeh	0.62	0.08	0.30
Karrubi	0.41	0.37	0.22
Moein	0.24	0.52	0.24
Abstention	0.01⋆⋆	0.01 ⋆⋆	0.99

Table 2: Original Goodman's regression of voting change between the two rounds

There are several problems related to the above results that is common among results drawn from aggregate data which is known as "ecological inference", specially those based on Goodman's regression. As King (1997) points out the first problem is that for the estimator to be unbiased in this linear model, parameters must be independent of the independent variables. For example, the coefficient of q_{AhAh}, which represents the probability that a person who voted for Ahmadinejad voting

for him again, must be independent of the proportion of population in a district who voted for Ahmadinejad in a district. The scatter graph below shows the data does not confirm that the two proportions are independent, and the estimator might be biased.
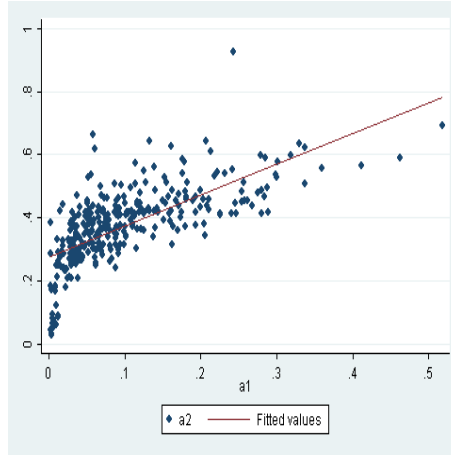

Figure 1: Ahmadinejad's voting share in the second round vs. the first round

Moreover, the coefficients are supposed to be between zero and one. In other words the estimator does not use this additional information. To improve the above result I use assumptions which are confirmed by the data to improve the results step by step. Making these step by step assumptions enable me to use the information on the data to increase the accuracy of the estimator. Below I state my first assumption:

Assumption 1. *Every body who voted for Ahmadinejad in the first round would vote for him in the second round.*

This assumption is consistent with the above results. Moreover it restricts the parameter q_{AhAh} to one. The results based on this assumption changes to: Note that the numbers in every row should add up to one since whatever the person has done in the first round, she has only three choice in the second round: vote for Ahmadinejad, vote for Rafsanjani, or abstain. For the next step I make the following assumption:

Assumption 2. *Every body who voted for Rafsanjani in the first round, would not abstain in the second round.*

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[3]not significant in %90 interval
[3]not significant in %99 interval

	Ahmadinejad	Rafsanjani	Abstention
Rafsanjani	0.43	0.74	-0.17
Larijani	0.85	0.19	-0.04a
Ghalibaf	0.82	0.14	0.04b
Mehralizadeh	0.61	0.08x	0.31
Karrubi	0.41	0.37	0.22
Moein	0.24	0.52	0.25
Abstention	0.02x	-0.004c	0.98

Table 3: First step estimates of voting change

Base on this assumption $q_{RaAb} = 0$. Then to estimate the model I used seemingly unrelated regression since the three equations have different right hand side variables now. The result is then:

	Ahmadinejad	Rafsanjani	Abstention
Rafsanjani	0.31	0.69	0y
Larijani	0.91	0.21	-0.13
Ghalibaf	0.89	0.16	-0.06b
Mehralizadeh	0.62	0.08x	0.29
Karrubi	0.43	0.37	0.19
Moein	0.25	0.52	0.23
Abstention	0.03x	-0.003c	0.98

Table 4: The second step estimates with $q_{RaAb} = 0$

The two previous assumptions were completely making sense, they were both logical and consistent with the data. However for the next assumption I mostly use the results reported above and the general information that we have about the candidates and their supporters.

Assumption 3. *Every body who voted for Larijani in the first round would not abstain in the second round.*

The above assumption is consistent with the general view about the supporter of Larijani who were mostly among conservative, more traditional part of the society. It also closer to what we have seen so far in our estimates in previous steps. The next table summarizes the results after this assumption.

And similarly for the next step I use the following assumption:

	Ahmadinejad	Rafsanjani	Abstention
Rafsanjani	0.31	0.70	0y
Larijani	0.82	0.18	0y
Ghalibaf	0.90	0.16	-0.08x
Mehralizadeh	0.62	0.08x	0.30
Karrubi	0.44	0.37	0.19
Moein	0.25	0.52	0.24
Abstention	0.03x	-0.002c	0.97

Table 5: The third step estimates with $q_{LaAb} = 0$

	Ahmadinejad	Rafsanjani	Abstention
Rafsanjani	0.32	0.70	0y
Larijani	0.82	0.18	0y
Ghalibaf	0.84	0.15	0y
Mehralizadeh	0.62	0.08x	0.29
Karrubi	0.44	0.38	0.17
Moein	0.25	0.53	0.22
Abstention	0.04x	0.0001c	0.96

Table 6: The Final step estimates with $q_{GhAb} = 0$

Assumption 4. *Every body who voted for Ghalibaf in the first round would not abstain in the second round.*

The argument is similar to what I did for the Assumption (3) and the result is:

The above results are consistent with the restrictions on the coefficients and show how people changed their votes between the two round. These results exhibit interesting aspects of the voting behavior in the 2005 election. One of the most interesting aspects is that a big proportion of the voters who voted for Rafsanjani in the first round (about 32 percent) did not vote for him again in the second round, instead voted for his opponent. The reason could be that these voters preferred Ahmadinejad over Rafsanjani in the first round too, but they did not vote for him because they did not believe him being a viable candidate.

Moreover Rafsanjani was unsuccessful to attract any significant vote from those who abstained in the first round. The results show that Ahmadinejad was able to capture the vote of about 4 percent of those potential voters, even though the coefficient is not significant in the %1 interval for error. That suggests that most of

those who abstained in the first round did it due to alienation, but those 4 percent who decided to change their decision to vote for Ahmadinejad abstained in the first round due to indifference; For them Ahmadinejad was a favorable candidate but there was not enough contrast between him and the other candidates in the first round. However, in the second round two things happened: they updated their perception about the likelihood of Ahmadinejad winning the election, and they updated the likelihood of Rafsanjani's chance as the only other alternative. Since Rafsanjani was an unfavorable candidate for them they were motivated enough to show up for voting.

Appendix

Here I present the proof for Proposition 4. I prove it by contradiction: Suppose the opposite is true; i.e. , $k \in \bar{\omega}$ and $l \notin \bar{\omega}$ but $u_i(x_l) > u_i(x_k)$.

Suppose we construct the set ω_l by adding the candidate l to the set $\bar{\omega}$ and ω_k by deleting k from $\bar{\omega}$. Since $\bar{\omega}$ maximizes the difference in the expected utility from a subset of candidates conditioned on one of them being elected and the expected utility from all other candidates conditioned on one of them being elected then:

$$\sum_{j \in \bar{\omega}} \frac{p_{ij} u_i(x_j)}{\sum_{j \in \bar{\omega}} p_{ij}} - \sum_{j \notin \bar{\omega}} \frac{p_{ij} u_i(x_j)}{\sum_{j \notin \bar{\omega}} p_{ij}} \geq \sum_{j \in \omega_l} \frac{p_{ij} u_i(x_j)}{\sum_{j \in \omega_l} p_{ij}} - \sum_{j \notin \omega_l} \frac{p_{ij} u_i(x_j)}{\sum_{j \notin \omega_l} p_{ij}}, \tag{9}$$

and

$$\sum_{j \in \bar{\omega}} \frac{p_{ij} u_i(x_j)}{\sum_{j \in \bar{\omega}} p_{ij}} - \sum_{j \notin \bar{\omega}} \frac{p_{ij} u_i(x_j)}{\sum_{j \notin \bar{\omega}} p_{ij}} \geq \sum_{j \in \omega_k} \frac{p_{ij} u_i(x_j)}{\sum_{j \in \omega_k} p_{ij}} - \sum_{j \notin \omega_k} \frac{p_{ij} u_i(x_j)}{\sum_{j \notin \omega_k} p_{ij}}. \tag{10}$$

But the right hand side of (9) can be rewritten as:

$$\frac{\sum_{j \in \bar{\omega}} p_{ij} u_i(x_j) + p_{il} u_i(x_{il})}{\sum_{j \in \bar{\omega}} p_{ij} + p_{il}} - \frac{\sum_{j \notin \bar{\omega}} p_{ij} u_i(x_j) - p_{il} u_i(x_{il})}{\sum_{j \notin \bar{\omega}} p_{ij} - p_{il}} \tag{11}$$

,

and similarly for 10:

$$\frac{\sum_{j \in \bar{\omega}} p_{ij} u_i(x_j) - p_{ik} u_i(x_{il})}{\sum_{j \in \bar{\omega}} p_{ij} - p_{ik}} - \frac{\sum_{j \notin \bar{\omega}} p_{ij} u_i(x_j) + p_{ik} u_i(x_{ik})}{\sum_{j \notin \bar{\omega}} p_{ij} + p_{ik}}. \tag{12}$$

By placing 11 as the right hand side of (9) and rearranging the inequality, and doing similar with (12) and (10), we can get the following two inequalities:

$$\frac{\sum_{j\in\bar{\omega}} p_{ij}u_i(x_j)}{(\sum_{j\in\bar{\omega}} p_{ij}+p_{il})\sum_{j\in\bar{\omega}} p_{ij}}+\frac{\sum_{j\notin\bar{\omega}} p_{ij}u_i(x_j)}{(\sum_{j\notin\bar{\omega}} p_{ij}-p_{il})\sum_{j\in\bar{\omega}} p_{ij}} > \frac{u_i(x_{il})}{(\sum_{j\in\bar{\omega}} p_{ij}+p_{il})(\sum_{j\notin\bar{\omega}} p_{ij}-p_{il})}$$

$$\frac{\sum_{j\in\bar{\omega}} p_{ij}u_i(x_j)}{(\sum_{j\in\bar{\omega}} p_{ij}-p_{ik})\sum_{j\in\bar{\omega}} p_{ij}}+\frac{\sum_{j\notin\bar{\omega}} p_{ij}u_i(x_j)}{(\sum_{j\notin\bar{\omega}} p_{ij}+p_{ik})\sum_{j\in\bar{\omega}} p_{ij}} < \frac{u_i(x_{ik})}{(\sum_{j\in\bar{\omega}} p_{ij}-p_{ik})(\sum_{j\notin\bar{\omega}} p_{ij}+p_{il})}.$$

And by rearranging the above two inequalities:

$$\frac{\sum_{j\in\bar{\omega}} p_{ij}u_i(x_j)}{\sum_{j\in\bar{\omega}} p_{ij}} - u_i(x_{il}) > u_i(x_{il}) - \frac{\sum_{j\notin\bar{\omega}} p_{ij}u_i(x_j)}{\sum_{j\notin\bar{\omega}} p_{ij}}, \tag{13}$$

and

$$\frac{\sum_{j\in\bar{\omega}} p_{ij}u_i(x_j)}{\sum_{j\in\bar{\omega}} p_{ij}} - u_i(x_{ik}) < u_i(x_{ik}) - \frac{\sum_{j\notin\bar{\omega}} p_{ij}u_i(x_j)}{\sum_{j\notin\bar{\omega}} p_{ij}}. \tag{14}$$

As the result, $u_i(x_{ik}) > u_i(x_{il})$ which is contradictory with our earlier assumption.

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